On-Line Appendix with Proofs for

"Does International Mobility of High-Skilled Workers Aggravate Between-Country Inequality?"

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Proof of Lemma 1. Factor prices in the final goods sector equal marginal productivities; thus, $w_H = \beta Y/H_Y$ and $w_L = (1-\alpha-\beta)Y/L_Y$. Using (7), we find that the relative input of the two types of labor in the domestic economy is independent of the level of migration:

$$\frac{H_Y}{L_Y} = \frac{\beta(1-e)}{1-\alpha-\beta}. \tag{28}$$

From the inverse demand function of any intermediate good firm j, optimal price $p(j) = r/\alpha$ and the production function in the final goods sector (3) we find

$$x_{t-1}(j) = \left(\frac{\alpha^2}{(1+r)r}\right)^{\frac{1}{1-\alpha}} \left(\frac{H_{Y,t}}{L_{t,Y}}\right)^{\frac{\beta}{1-\alpha}} A_t L_{Y,t} \equiv \bar{x}_{t-1}. \tag{29}$$

The production function for the capital good, (4), implies that $(X_t)^\alpha = n_{t-1}(\bar{x}_{t-1})^\alpha$ Substituting this into (3) and using both (28) and (29) leads to

$$Y_t = n_{t-1} \left(\frac{\alpha^2}{(1+r)r}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\beta(1-e)}{1-\alpha-\beta}\right)^{\frac{\beta}{1-\alpha}} A_t L_{Y,t}. \tag{30}$$

Substituting (30) into $w_L = (1-\alpha-\beta)Y/L$ and combining the resulting expression with $w_H = \frac{w_L}{1-e}$ from (7) confirms (14).

Expression (16) follows from substituting (14) into (13) and using $w_H^{net} = (1-\tau)w_H$.

To confirm (15), first, insert $H = 1-L$ in (12) and use both (28) and (11) to find

$$L_Y = L = \frac{1-\alpha-\beta}{1-\alpha}\left(1-m-\frac{nf}{1-e}\right). \tag{31}$$

Next, we employ the zero-profit condition for intermediate good firms, $\pi_{t-1}(j) = 0$, or $(p_{t-1}(j) - r)\, x_{t-1}(j) = \frac{w_{H,t}f}{1+r}$, according to (10). Substituting into the latter equation $p_{t-1}(j) = r/\alpha$, the expression for $x_{t-1}(j)$ in (29) and the expression for $w_{H,t}$ in (14), as well as using (28) and (31) leads to

$$\alpha(1-e)\left(1-m_t-\frac{n_t f}{1-e}\right)=n_{t-1}f. \tag{32}$$

Substituting (16) into (32) and solving for n_t we obtain (15). From (15), it it is straightforward to derive the claimed properties of function $Z(n)$. ■

Proof of Proposition 1. According to Lemma 1, function Z starts at zero and initially has a slope above unity which eventually turns negative. Because we know that, in addition, $Z'' < 0$ holds under (A1), there is a single non-zero value n^* which fulfills $Z(n^*) = n^*$. This confirms part (i). To confirm part (ii), note from the definition of Z in (15) that the value of Z decreases for each $n > 0$ if mobility costs decline and employ Fig. 1. Part (iii) can immediately be inferred from Fig. 1. ■

Proof of Proposition 2. First, note from the definition of G_1 that it is given by $w_G(G_1, n_1) = 0$ (recall $w_{GG} < 0$). Recalling $n_1 < n_0$ and $w_{Gn} > 0$ confirms (24).

In steady state, the first-order condition to the maximization of $W(G,\chi)$ reads

$$[W_G(G,\chi) =]\, w_G(G,\hat{n}(G,\chi)) + w_n(G,\hat{n}(G,\chi))\hat{n}_G(G,\chi) = 0, \tag{33}$$

according to (22). Applying the implicit function theorem to (21), we obtain:

$$\hat{n}_G(G,\chi) = \frac{\Gamma_G(G,\hat{n},\chi)}{1-\Gamma_n(G,\hat{n},\chi)}. \tag{34}$$

Note that the denominator is positive in a stable steady state equilibrium ($\Gamma_n(G,\hat{n},\chi) < 1$). Moreover, from (20) we find

$$\Gamma_G(G,\hat{n},\chi) = -\frac{1-e}{f}\chi q'\left(w(G,\hat{n})\right) w_G(G,\hat{n}). \tag{35}$$

Substituting (34) into (33) and using (35), we can rewrite the first-order condition to

$$w_G(G,\hat{n})\left[1-\frac{1-e}{f}\chi q'\left(w(G,\hat{n})\right)\frac{w_n(G,\hat{n})}{1-\Gamma_n(G,\hat{n},\chi)}\right] = 0. \tag{36}$$

Define $G^*(\chi) \equiv \arg\max_G W(G,\chi)$ as the optimal log-run public investment level. Suppose that G^* is given by first-order condition (33). (It will become apparent that the second-order condition indeed holds.) As the term in squared brackets in (36) is positive, we find that G^* is given by

$$w_G(G^*, \hat{n}(G^*,\chi)) = 0. \tag{37}$$

Thus, we also have $\hat{n}_G(G^*,\chi) = 0$, according to (34) and (35).

We next show that the second-order condition holds, i.e., $W_{GG}(G^*,\chi) < 0$. To see this, note that $\hat{n}_G(G^*,\chi) = 0$ implies that $W_G(G^*,\chi) = w_G(G^*, \hat{n}(G^*,\chi))$ when G^* is given by first-order condition (33). Hence,

$$W_{GG}(G^*,\chi) = \left.(w_{GG} + w_{Gn}\hat{n}_G)\right|_{G=G^*}. \tag{38}$$

Using again $\hat{n}_G(G^*,\chi) = 0$, we thus have $W_{GG}(G^*,\chi) = \left.(w_{GG})\right|_{G=G^*}$. Recalling that $w_{GG} < 0$ confirms that the second-order condition holds.

Moreover, we have

$$W_{G\chi}(G^*,\chi) = \left.(w_{Gn}\hat{n}_\chi)\right|_{G=G^*}. \tag{39}$$

Thus,

$$\frac{\mathrm{d}G^*(\chi)}{\mathrm{d}\chi} = -\frac{W_{G\chi}(G^*,\chi)}{W_{GG}(G^*,\chi)} = \left.\left(-\frac{w_{Gn}\hat{n}_\chi}{w_{GG}}\right)\right|_{G=G^*}. \tag{40}$$

Since $w_{GG} < 0$, $w_{Gn} > 0$ and $\hat{n}_\chi < 0$, we find that G^* is decreasing with χ, which confirms (25) and concludes the proof of part (i).

To prove part (ii), recall first that $w_n > 0$. Since public investment is chosen optimally before and after the change in the degree of labor market integration ($w_G(G_1, n_1) = w_G(G_0, n_0) = 0$) and $n_1 < n_0$, we have $w(G_1, n_1) < w(G_0, n_0)$ for the net wage rate. For the level of emigration, according to (19) and property $q' < 0$, this implies

$$m_1 = \chi_1 q(w(G_1, n_1)) > \chi_0 q(w(G_0, n_0)) = m_0. \tag{41}$$

This confirms the result for the subsequent period after labor market integration.

Now write $G^*(\chi)$ as the function which is implicitly defined by $\hat{n}_G(G^*,\chi) = 0$ and define $W^*(\chi) \equiv W(G^*(\chi), \chi)$. $W^*(\chi)$ is the steady state value of the net wage rate w_H^{net} when G is chosen optimally. We find that

$$\frac{\mathrm{d}W^*}{\mathrm{d}\chi} = W_G(G^*, \chi)\frac{\mathrm{d}G^*}{\mathrm{d}\chi} + W_\chi(G^*, \chi). \tag{42}$$

Note that $W_G(G^*, \chi) = 0$ and $W_\chi(G^*, \chi) = w_n \hat{n}_\chi|_{G=G^*} < 0$, where the latter is implied from using definition (22) together with $\hat{n}_G(G^*, \chi) = 0$. Thus, $\mathrm{d}W^*/\mathrm{d}\chi < 0$. Moreover, note that the steady state number of migrants is given by

$$m^*(\chi) \equiv \chi q(W^*(\chi)), \tag{43}$$

under the optimal choice of G. Using $q' < 0$ then implies that m^* is increasing in χ. This concludes the proof. ■

Proof of Corollary 1. Using (34) and (35) together with $w_G(G^*, \hat{n}) = \hat{n}_G(G^*, \chi) = 0$, it is easy to confirm that $\hat{n}_{GG}(G^*, \chi) < 0$, by utilizing property $w_{GG} < 0$. This shows that G^*, which is given by (37), maximizes $\hat{n}(G, \chi)$. ■

Proof of Proposition 3. Analogously to (20), by using (26), the difference equation for the evolution of n can be written as

$$n_t = \frac{1-e}{f}\left[1 - \chi q(\tilde{w}(G_t, B_t, n_{t-1}))\right] - \frac{n_{t-1}}{\alpha} \equiv \tilde{\Gamma}(G_t, B_t, n_{t-1}, \chi). \tag{44}$$

For a given fiscal policy, (G, B), the steady state number of firms, n^*, is implicitly defined by $n^* = \tilde{\Gamma}(G, B, n^*, \chi)$, where stability requires that $\tilde{\Gamma}_n(G, B, n^*, \chi) < 1$. n^* is a function of (G, B, χ) which is denoted by $\tilde{n}(G, B, \chi)$. Substituting $m = \chi q(w_H^{net})$ and (26) into (27), long run welfare can be written as

$$W(G, B, \chi) \equiv v((1-e)\tilde{w}(G, B, \tilde{n}(G, B, \chi))) - \xi(\chi q(\tilde{w}(G, B, \tilde{n}(G, B, \chi)))) - \psi(B). \tag{45}$$

Since the economy is initially in a stable steady state, initially, fiscal policy is given by

$$(G_0^*, B_0^*) \equiv \arg \max_{(G,B)} W(G, B, \chi_0). \tag{46}$$

Thus, the initial number of firms is $n_0 = \tilde{n}(G_0^*, B_0^*, \chi_0) = \tilde{\Gamma}(G_0^*, B_0^*, n_0, \chi_0)$. Moreover, if labor market integration shifts from χ_0 to $\chi_1 > \chi_0$, we have $n_1 = \tilde{\Gamma}(G_0^*, B_0^*, n_0, \chi_1) < n_0$, according to (44). Also define

$$\bar{W}(G, B, n) \equiv v((1-e)\tilde{w}(G, B, n)) - \xi(\chi q(\tilde{w}(G, B, n))) - \psi(B) \tag{47}$$

and

$$(G_1, B_1) \equiv \arg \max_{(G,B)} \bar{W}(G, B, n_1). \tag{48}$$

First-order conditions to the maximization problem in (48) are:

$$\bar{W}_G = [v'((1-e)\tilde{w})(1-e) - \xi'(\chi q(\tilde{w}))\chi q'(\tilde{w})]\, \tilde{w}_G = 0, \tag{49}$$

$$\bar{W}_B = [v'((1-e)\tilde{w})(1-e) - \xi'(\chi q(\tilde{w}))\chi q'(\tilde{w})]\, \tilde{w}_B - \psi'(B) = 0. \tag{50}$$

Since the term in squared brackets in (49) and (50) is positive (recall $v' > 0$, $\xi' > 0$, $q' < 0$), we have $\tilde{w}_G(G_1, B_1, n) = 0$. Together with $\tilde{w}_{GB} = 0$, we thus find that $\bar{W}_{GB}(G_1, B_1, n_1) = 0$. Moreover, $\bar{W}_{GG}(G_1, B_1, n_1) < 0$ and $\bar{W}_{Gn}(G_1, B_1, n_1) > 0$ since $\tilde{w}_{GG} < 0$ and $\tilde{w}_{Gn} > 0$, respectively. Thus, G_1 is decreasing in n_1. Since $n_1 < n_0$, it follows that $G_1 < G_0^*$. Moreover, we have

$$\bar{W}_{BB} = \left[(1-e)^2 v'' - \xi''(\cdot)\chi^2 q' - \xi' \chi q''\right] (\tilde{w}_B)^2 - \psi'' < 0, \tag{51}$$

$$\bar{W}_{Bn} = \left[(1-e)^2 v'' - \xi''(\cdot)\chi^2 q' - \xi' \chi q''\right] \tilde{w}_n \tilde{w}_B < 0 \tag{52}$$

(recall $\tilde{w}_{BB} = 0$, $v'' < 0$, $\xi'' \geq 0$, $q'' > 0$, $\psi'' \geq 0$). Thus, B_1 is decreasing in n_1. Since $n_1 < n_0$, it follows that $B_1 > B_0^*$.

It remains to be shown that long run effects are ambiguous. The first-order conditions to the problem of maximizing long run welfare (45)), $W(G, B, \chi)$, with respect to (G, B), are:

$$\begin{aligned} W_G &= \left[v'((1-e)\tilde{w})(1-e) - \xi'(\chi q(\tilde{w}))\chi q'(\tilde{w})\right](\tilde{w}_G + \tilde{w}_n \tilde{n}_G) = 0, \qquad (53) \\ W_B &= \left[v'((1-e)\tilde{w})(1-e) - \xi'(\chi q(\tilde{w}))\chi q'(\tilde{w})\right](\tilde{w}_B + \tilde{w}_n \tilde{n}_B) - \psi'(B) = 0. \qquad (54) \end{aligned}$$

According to (44), we have

$$\tilde{n}_G = -\frac{\frac{1-e}{f}\chi q'(\tilde{w})\tilde{w}_G}{1-\tilde{\Gamma}_n} \text{ and } \tilde{n}_B = -\frac{\frac{1-e}{f}\chi q'(\tilde{w})\tilde{w}_B}{1-\tilde{\Gamma}_n} > 0. \qquad (55)$$

The latter inequality follows from $\tilde{\Gamma}_n < 1$ (which holds in stable steady state), $q' < 0$ and $\tilde{w}_B > 0$. Using (55) in (53) and (54), we can write

$$\begin{aligned} W_G &= \Omega\Theta\tilde{w}_G = 0, \qquad (56) \\ W_B &= \Omega\Theta\tilde{w}_B - \psi'(B) = 0, \qquad (57) \end{aligned}$$

where

$$\begin{aligned} \Omega &\equiv \left.\left[v'((1-e)\tilde{w})(1-e) - \xi'(\chi q(\tilde{w}))\chi q'(\tilde{w})\right]\right|_{n=n^*}, \qquad (58) \\ \Theta &\equiv \left.\left(1 - \frac{\frac{1-e}{f}\chi q'(\tilde{w})}{1-\tilde{\Gamma}_n}\right)\right|_{n=n^*}. \qquad (59) \end{aligned}$$

Note that $\Omega > 0$ and $\Theta > 0$. Thus, at the optimal long run levels (G^*, B^*), it holds that $\tilde{w}_G = \tilde{n}_G = 0$. This implies

$$\begin{aligned} W_{GG}|_{(G^*,B^*)} &= \Omega\Theta\tilde{w}_{GG} < 0, \qquad (60) \\ W_{GB}|_{(G^*,B^*)} &= \Omega\Theta(\tilde{w}_{GB} + \tilde{w}_{Gn}\tilde{n}_B) > 0, \qquad (61) \end{aligned}$$

where the inequality in (60) follows from $\tilde{w}_{GG} < 0$ and the one in (61) from $\tilde{w}_{GB} = 0$, $\tilde{w}_{Gn} > 0$, $\tilde{n}_B > 0$. Moreover,

$$W_{G\chi}|_{(G^*,B^*)} = \Omega\Theta\tilde{w}_{Gn}\tilde{n}_\chi < 0, \qquad (62)$$

where the inequality follows from $\tilde{w}_{Gn} > 0$ and

$$\tilde{n}_\chi = -\frac{\frac{1-e}{f}q(\tilde{w})}{1-\tilde{\Gamma}_n} < 0. \tag{63}$$

Next, note that

$$\frac{\partial\Omega}{\partial B} = \left[(1-e)^2 v'' - \xi''\chi^2 q'(\cdot)^2 - \xi'\chi q''(\cdot)\right](\tilde{w}_B + \tilde{w}_n\tilde{n}_B), \tag{64}$$

$$\frac{\partial\Omega}{\partial\chi} = (1-e)^2 v''\tilde{w}_n\tilde{n}_\chi - \xi''\chi q'\left[q + \chi q'\tilde{w}_n\tilde{n}_\chi\right] - \xi'\left[q' + \chi q''\tilde{w}_n\tilde{n}_\chi\right], \tag{65}$$

$$\frac{\partial\Theta}{\partial B} = -\frac{1-e}{f}\chi\frac{q''(\tilde{w}_B + \tilde{w}_n\tilde{n}_B)(1-\tilde{\Gamma}_n) + q'(\tilde{\Gamma}_{nB} + \tilde{\Gamma}_{nn}\tilde{n}_B)}{\left(1-\tilde{\Gamma}_n\right)^2}, \tag{66}$$

$$\frac{\partial\Theta}{\partial\chi} = -\frac{1-e}{f}\frac{\left[q' + \chi q''\tilde{w}_n\tilde{n}_\chi\right](1-\tilde{\Gamma}_n) + \chi q'(\tilde{\Gamma}_{n\chi} + \tilde{\Gamma}_{nn}\tilde{n}_\chi)}{\left(1-\tilde{\Gamma}_n\right)^2}. \tag{67}$$

From $v'' < 0$, $\xi'' \geq 0$, $q' < 0$, $\xi' \geq 0$, $q'' > 0$, $\tilde{w}_B > 0$, $\tilde{w}_n > 0$, $\tilde{n}_B > 0$, $\tilde{n}_\chi < 0$, we find that $\partial\Omega/\partial B < 0$ and $\partial\Omega/\partial\chi > 0$. Moreover, using the definition of $\tilde{\Gamma}$ in (44), we have

$$\tilde{\Gamma}_n = -\frac{1-e}{f}\chi q'(\tilde{w})\tilde{w}_n - \frac{1+r}{\alpha}.$$

Thus, recalling $\tilde{w}_B > 0$, $\tilde{w}_n > 0$, $\tilde{w}_{nn} = 0$, $\tilde{n}_B > 0$, $\tilde{n}_\chi < 0$, $q' < 0$, $q'' > 0$, we find

$$\tilde{\Gamma}_{nB} = -\frac{1-e}{f}\chi q''(\tilde{w}_B + \tilde{w}_n\tilde{n}_B) < 0, \tag{68}$$

$$\tilde{\Gamma}_{nn} = -\frac{1-e}{f}\chi\left[q''(\tilde{w}_n)^2 + q'\tilde{w}_{nn}\right] < 0, \tag{69}$$

$$\tilde{\Gamma}_{n\chi} = -\frac{1-e}{f}q'\tilde{w}_n > 0. \tag{70}$$

Thus, $\partial\Theta/\partial B < 0$ and $\partial\Theta/\partial\chi > 0$. Recalling $\tilde{w}_{BB} = \tilde{w}_{Bn} = 0$ and $\psi'' \geq 0$, we then have

$$W_{BB} = \frac{\partial\Omega}{\partial B}\Theta\tilde{w}_B + \Omega\frac{\partial\Theta}{\partial B}\tilde{w}_B + \Omega\Theta(\tilde{w}_{BB} + \tilde{w}_{Bn}\tilde{n}_B) - \psi''(B) < 0, \tag{71}$$

$$W_{B\chi} = \frac{\partial\Omega}{\partial\chi}\Theta\tilde{w}_B + \Omega\frac{\partial\Theta}{\partial\chi}\tilde{w}_B + \Omega\Theta\tilde{w}_{Bn}\tilde{n}_\chi > 0. \tag{72}$$

Note that concavity of W as a function of (G, B) requires that $W_{GG}W_{BB} - (W_{GB})^2 >$

0. According to Cramer's rule, we then have

$$sgn\left(\frac{\mathrm{d}G^*}{\mathrm{d}\chi}\right) = sgn\left(-W_{G\chi}W_{BB} + W_{BG}W_{B\chi}\right)|_{(G^*,B^*)}, \tag{73}$$

$$sgn\left(\frac{\mathrm{d}B^*}{\mathrm{d}\chi}\right) = sgn\left(-W_{GG}W_{B\chi} + W_{GB}W_{G\chi}\right)|_{(G^*,B^*)}. \tag{74}$$

Thus, using the previous results on the signs of the second derivatives on the right hand sides of (73) and (74) confirms that long run effects of labor market integration on fiscal variables are ambiguous. This confirms part (i).

To prove part (ii), first note that analogously to the proof of part (ii) of Proposition 2, $n_1 < n_0$ implies that w_H^{net} declines and m increases in the period subsequent to labor market integration. To show the result for the steady state, define $W^*(\chi) \equiv W(G^*(\chi), B^*(\chi), \chi)$. We find that

$$\frac{\mathrm{d}W^*}{\mathrm{d}\chi} = W_G(G^*, B^*, \chi)\frac{\mathrm{d}G^*}{\mathrm{d}\chi} + W_B(G^*, B^*, \chi)\frac{\mathrm{d}B^*}{\mathrm{d}\chi} + W_\chi(G^*, B^*, \chi), \tag{75}$$

where $W_G = W_B = 0$ at (G^*, B^*) and, according to (45),

$$W_\chi = (1-e)v'\tilde{w}_n\tilde{n}_\chi - \xi'\left[q + \chi q'\tilde{w}_n\tilde{n}_\chi\right] < 0. \tag{76}$$

Thus, $\mathrm{d}W^*/\mathrm{d}\chi < 0$. Analogously to the proof of Proposition 2, together with (43) for the steady state level of migration, this concludes the proof. ■